A Preferential DL Approach to Model the Non bis in idem Principle for the Legal Domain

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\textbf{Abstract.} Description Logics (DLs) are a family of formalisms that emerged to balance the trade-off between expressiveness and decidability for classical monotonic logic. Often, the research developed under the umbrella of AI \& Law has relied on full synergy with DL to support argumentation reasoning, decision systems, legal compliance checking, and axiomatization of rationales and assumptions in the legal domain. Nevertheless, in many legal scenarios, regulations are defeasible. Inferences within the legal field are not purely deductive in nature, but retractable and ampliative, since generalizations mostly hold for normal or typical cases. This is absolutely true in the criminal domain, where general criminal types are usually described in the caput of the norms (e.g., a robbery), and other specific types unfold from these (e.g., robbery followed by death, which is known as “Latrocínio” in Brazilian Criminal law). Although the classical subsumption relation may seem a correct way to model the hierarchy of laws at first glance, if no contradiction arises between the more general and more specific, what it should be pointed out that the penalty for specific crimes cancel out the penalties foreseen by the more general laws. In other words, a hierarchy of norms must not rely on classical subsumption relation; instead, a non-monotonic approach suits better in this setting. Therefore, in this paper, we show that Preferential DL, a defeasible version of Description Logic, is better suited than classical DLs for a faithful representation of the content of legal regulatory knowledge; in particular, w.r.t. the representation of the principle of Double Jeopardy (a.k.a. Non/Ne bis in idem). In this paper, we make the case for the application of ontologies represented in defeasible, preferential DLs for modelling laws and penalties. Our solution focuses on overrule relations to organize a set of defeasible axioms in terms of specificity criteria.

\textbf{Keywords:} Description Logic, Non-monotonic Reasoning, Legal Domain, Bis in idem, Double Jeopardy

\section{Introduction}

As time passes, legislative chambers around the world have gradually engineered and upgraded their legal systems, reacting to social, cultural, economic, and political changes. Legal regulations need to consistently accommodate such changes, which, in general, do not follow a systematized protocol, leading to potential inconsistencies in the system as a whole. Legal provisions (i.e., what is stated in a legal norm) use to be directed exclusively for trained professionals, i.e., judges and attorneys’ consumption. At present, as government data is becoming openly available, the legal domain should serve a wider audience. Concerns focus on tackling methodologies to represent legal rules, case interpretations and practitioner insights.

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into both human (professional or laymen people) and machine-readable models.

In the early years of the last decade, we witnessed a phenomenon – the Semantic Web [1] – aimed at extending the traditional web with semantically annotated resources. The underlying logical layer of the new web architecture [2] allows to axiomatize entities, types and roles of arbitrary domains through fine-grained conceptual models. Logical inference capacity became equally possible, opening new possibilities not yet envisioned, particularly for the legal domain. Since then, the related literature has experienced an explosion of logical models, of the most diverse realms, functions, and ontological engagements. Additionally, a synergetic relationship between the Semantic Web and Ontologies has been observed; the latter served to strengthen the traditional web with semantically annotated resources. The underlying logical layer of the web architecture [2] allows to axiomatize entities, types and roles of arbitrary domains through fine-grained conceptual models.

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Although several studies look at the legal domain as an opportune landscape for Semantic-Web applications [5–7], the representation of legal knowledge faces semantic and syntactic anomalies, such as normative conflicts. According to Lindahl (1992) [8], a normative conflict occurs in a situation where it is impossible to apply two rules together. Boer et al. (2005) [9] list three principal normative conflicts: Dis-affirmation conflict (two different deontic mode norms in opposition), Compliance conflict (two norms with the same deontic mode remain opposed), and conflicts between defeasible rules. In this paper, we are particularly interested in the latter type, which prevents, for example, situations in which one ought to do substantially the same thing twice in different ways, introducing a violation of the globally accepted legal principle known as Double Jeopardy.

The Double Jeopardy principle (a.k.a. the maxim non/ne bis in idem [10]) accommodates the widely adopted legal rule (it is part of the universal law of nations [11]) that an individual should only be punished once for one offense, avoiding a disproportionate punishment to the offender. That is, the principle poses situations susceptible to defeasible reasoning.

For instance, suppose a peculiar situation related to the Brazilian Criminal Law. A person A deliberately steals the wallet from person B; without further information, the situation typically constitutes a Theft (art. 155 of Brazilian penal code). Additional circumstances, such as stealing followed by aggression (serious threat or violence) would draw a more serious crime, Robbery (art. 157 from the Brazilian penal code), that might lead to more severe penalties. In other words, the former inference remains unchanged until the insertion of specific circumstances triggers a more exceptional case (e.g., robbery), whose penalty should replace the one from the more general case and removes the previous conclusion. From the caput defining robbery, other more severe types are built within this, such as robbery followed by death (“latrocínio”, art. 157. § 3.). Even so, courts of law tend to rule out robbery classification, particularly when the crime of “latrocínio” occurs in the attempted mode (the death of the victim did not materialize, even if the perpetrator sought a fatal action). If it were not so, defense lawyers would conveniently request the reclassification of the crime of attempted “robbery followed by death” as robbery only.

What is going on in the aforementioned cases is that we might have two legal rules that apply to the same facts, but only one rule should be applied. Moreover, the rules have a “hierarchy”, where we should select the superior one. As a result, it should be noted that classical logic, which is the basis of DL, does not fully fit into all juridical nuances [12]. Therefore, classical DL may lead to problems or solutions that do not properly represent the usual human behavior tackled in the aforementioned situation. On the other hand, Preferential DL [13] is a non-monotonic super set of DL, which seems well-suited to represent related cases and laws in the legal field. Preferential DL is more adequate than its classical counterpart to model and reason with partial or incomplete information, thereby avoiding the violation of ordinary legal principles, such as the non bis in idem.

This work proposes an (onto)logical approach with defeasible axioms, instead of classical ones, to model the criminal types of Brazilian law that abides by the Double Jeopardy principle. Such an approach prevents undesirable results with respect to the judicial practice. Therefore, using the Preferential DL formalism and its theoretical basis [13], whenever an illicit behavior is classified into more than one criminal type, including different punishments, a decision support system may aptly infer the appropriate sanctions foreseen by the legislation. This decision naturally takes into account the criteria of specificity existing among those crimes. Indeed, the solution presented here addresses a proposal for preferential (and rational) overrule relations.

https://www.w3.org/OWL
to hierarchically organize a set of definitional axioms within an arbitrary legal knowledge base. With this solution at hand, penalties of the more specific axioms cancel out penalties of less specific ones.

The remainder of the paper is structured as follows. First, in Section 2 we provide the required background on classical description logic and its non-monotonic counterpart, preferential DLs. In Section 3, we pose the nuances of the legal universe and compare the classical and non-monotonic solutions to model legal domains, showing exactly where classical DL fails in such cases. In Section 4, we discuss how preferential DL offers more appropriate solutions to these settings. In addition, we present a general axiomatization for criminal norms in terms of defeasible axioms, along with a notion of overrule relations. In Section 5, related work is described. We conclude the paper along with a discussion about the results achieved so far and final remarks (Section 6).

2. Theoretical Background: Description Logic and Preferential DL

2.1. Description Logics

Description Logics (DLs) [4] are a family of knowledge representation formalisms, with support for reasoning tasks. DLs are a well-behaved fragment of L2, i.e., First-Order Logic (FOL) with 2 variables (decidable and NEXPTIME-complete), quite expressive and applicable to many areas. Several (and heterogeneous) potential applications require different levels of expressiveness and complexity of reasoning, which justifies the use of different DL fragments. Furthermore, these formalisms stand as the formal foundation of the Semantic Web endeavour.

In what follows, we describe the syntax and semantics of a reasonably expressive DL family member, SHIQ [14], and the associated reasoning tasks. SHIQ is the foundation for the well-known OWL language (Ontology Web Language). OWL constitutes the Ontology Layer in the original Semantic Web vision [1].

2.1.1. Syntax of SHIQ

DL sublanguages are described by the constructors they provide; each one dictates a trade-off between expressiveness and reasoning computational complexity. In particular, SHIQ [14] is still one of the most expressive yet decidable DLs. The syntax of SHIQ is arranged in terms of three pairwise disjoint finite sets of symbols: the set of concept names (NC), the set of role names (NR), and the set of nominals (NO). The grammar below defines the structure of SHIQ concept expressions:

\[
C, D ::= NC \mid C \sqcap C \mid C \sqcup C \mid \neg C \mid \top \mid \exists ! r.C \mid \forall r.C \mid n r.C \mid \{ N_0 \} \\
R, S ::= NR \mid NR^\perp
\]

While the first definition describes the set of SHIQ concept expressions, the second defines role expressions. We denote L as the set of SHIQ concepts. Concept constructors can be classified into Boolean constructs and relationship constraints. A DL knowledge base (KB) is structured in three components, KB := (A, T, R) (actually, with nominals, A becomes syntactic sugar, however, we keep it for reasons of clarity [14]), where:

- \(A\) is the assertional axioms, in which, we find (suppose a, b are individuals of NO):
  * Concept assertion, \(a : C\);
  * Role assertion, \((a, b) : R\);
  * Individual equality and inequality, \(a \approx b, a \not\approx b\), respectively;
- \(T\) is the terminological axioms, in which, we find:
  * Concept inclusion, \(C \subseteq D\);
  * Concept equivalence, \(C \equiv D\);
- \(R\) is the relational axioms, in which, we find:
  * Role inclusion, \(R \sqsubseteq S\);
  * Role equivalence, \(R \equiv S\);
  * Complex role inclusion, \(R_1 \circ R_2 \sqsubseteq R\);
  * Inverse roles, Invr(R) = R\(^{-}\);
  * Transitive roles, Trans(R) = (\(+\) R).

2.1.2. Semantics of SHIQ

DL semantics assumes the Open-World Assumption (OWA) [15]. According to the OWA premise, given a state-of-affairs, the absence of information is treated as something unknown, unlike the Closed-World assumption [15, 16], where it is interpreted as negative information. OWA is the most reliable assumption for the representation of knowledge, whenever it is not possible to guarantee that all information has been provided, or have not been made available yet.

Definition 2.1. Classical DL Interpretation. An interpretation is a tuple \( \mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I}) \), where \( \Delta^\mathcal{I} \) represents the non-empty set known as the domain of \( \mathcal{I} \), while \( \cdot^\mathcal{I} \) is a function that maps concepts to subsets of \( \Delta^\mathcal{I} \), relations to subsets of \( \Delta^\mathcal{I} \times \Delta^\mathcal{I} \) and instances to elements of \( \Delta^\mathcal{I} \).

In addition, in this same context, it is necessary to describe the definition of Model.

Definition 2.2. Model. An interpretation \( \mathcal{I} \) is a model of a concept \( C \) if \( C^\mathcal{I} \neq \emptyset \). In addition, an interpretation \( \mathcal{I} \) is a model of a KB := \( \langle \mathcal{A}, \mathcal{T} \rangle \) if \( \mathcal{I} \) is a model of \( \mathcal{T} \) and a model of \( \mathcal{A} \).

Table 1 highlights the self-explanatory semantics for DL grammar constructors and TBox/ABox/RBox axioms. For a short and didactic introduction to DL and its applications, as well as semantics for remaining constructors, check additional reference [4, 14].

2.1.3. Reasoning over Concept Expressions and Knowledge Bases

From the definition of Interpretation in DL, we can state some basic tasks of reasoning over concept expressions as such as:

- **Concept Satisfiability**: Given a concept \( C, C \) is satisfiable iff it admits a model.
- **Concept Subsumption**: An interpretation \( \mathcal{I} \) is a model of a general concept subsumption \( C \subseteq D \) if \( C^\mathcal{I} \subseteq D^\mathcal{I} \).

In addition, DLs have associated a number of reasoning tasks that are important from the standpoint of knowledge representation and reasoning. Among them are:

- **Knowledge Base Satisfiability**: Given a knowledge base KB, and two concepts \( C \) and \( D \), KB is satisfiable if it admits a model, that is, an Interpretation \( \mathcal{I} \), which for every axiom \( C \subseteq D \) in KB, \( C^\mathcal{I} \subseteq D^\mathcal{I} \).
- **Concept Satisfiability w.r.t. Knowledge Base** (KB \( \models C \subseteq D \)): A knowledge base KB, and a concept \( C, C \) is satisfiable w.r.t. KB if there is an interpretation \( \mathcal{I} \), which is a model for KB, and further a model for \( C, C \), that is, \( C^\mathcal{I} \neq \emptyset \).
- **Logical Implication** (KB \( \models C \subseteq D \)): Given a knowledge base KB, and two concepts \( C \) and \( D \), KB subsumes \( C \), if for all models \( \mathcal{I} \) of KB, \( C^\mathcal{I} \subseteq D^\mathcal{I} \).

The reasoning tasks described hitherto, however, address only the terminological portion (\( \mathcal{T} \)) of a knowledge base. Therefore, given individual names \( a, b \) in \( N^\mathcal{T} \), for the assertional component, we have:

- **Concept Instantiation** (KB \( \models a : C \)): Given a knowledge base KB, and an individual \( a, a \) is an instance of concept \( C \) w.r.t. KB if \( a^\mathcal{I} \in C^\mathcal{I} \) holds for all models \( \mathcal{I} \) of KB;
- **Role Name Instantiation** (KB \( \models (a, b) : R \)): Given a knowledge base KB, and some individuals \( a, b \), the pair of individuals \( (a, b) \) is an instance of role name \( R \) w.r.t. KB if \( \{a^\mathcal{I}, b^\mathcal{I}\} \in R^\mathcal{I} \) holds for all models \( \mathcal{I} \) of KB;

Horrocks (2005)[17] argues that those decidable key inference problems — concept satisfiability and logical subsumption — were the key to DL becoming the ba-
In order to leverage the DL semantics bearing non-monotonic reasoning, Britz et al. (2011) [13] suggest a DL extension addressing a defeasible subsumption constructor \( \sqsubseteq \) to axiomatize exceptions for the typical objects, known as Preferential Description Logic. Consider, for example, the universe of flying birds and penguins. Instead of stating, for example, that birds fly \((\text{Birds} \sqsubseteq \text{Fly})\), through a preferential DL syntax, we state that typical birds (the most base cases) fly \((\text{Birds} \sqsubseteq \text{Fly})\). In order to cope with exceptionality, \( \sqsubseteq \) should not be monotonic. Under such conditions, it is common to speak of a defeasible knowledge base, that is, one with a set of classical and a set of defeasible axioms. In the following, we highlight the preferential and rational subsumption relation defined by Britz et al. (2011) [13], and derived from Kraus et al. (1990) [24] (also known as KLM theory).

**Definition 2.3. (Preferential Subsumption [13]).** A subsumption relation \( \sqsubseteq \subseteq \mathcal{L} \times \mathcal{L} \) is a preferential subsumption relation iff it satisfies the following properties, where Ref stands for Reflexivity, LLE for Left Logical Equivalence, RW stands for Right Weakening, and CM stands for Cautious Monotonicity:

\[
\begin{align*}
\text{(Ref)} & \quad C \sqsubseteq C \quad \text{(LLE)} & \quad C \equiv D, C \sqsubseteq E & \quad D \sqsubseteq E \\
\text{(And)} & \quad C \sqsubseteq D, C \sqsubseteq E & \quad C \sqsubseteq D \sqcap E & \quad (\text{Or}) & \quad C \sqsubseteq E, D \sqsubseteq E & \quad C \sqcup D \sqsubseteq E \\
\text{(RW)} & \quad C \sqsubseteq D, D \sqsubseteq E & \quad C \sqsubseteq E & \quad (CM) & \quad C \sqsubseteq D, C \sqsubseteq E & \quad C \sqcap D \sqsubseteq E
\end{align*}
\]

**Definition 2.4. (Rational Subsumption [13]).** A subsumption relation \( \sqsubseteq \subseteq \mathcal{L} \times \mathcal{L} \) is a rational subsumption relation iff in addition to being a preferential subsumption relation, it also satisfies the Rational Monotonicity property (RM).

\[
\text{(RM)} \quad C \sqsubseteq D, C \nsubseteq \neg E \quad \quad \Rightarrow \quad \quad C \sqcap E \sqsubseteq D
\]

Extending DL with preferential/rational subsumption relations favors non-monotonic reasoning capabilities. For defeasible subsumption inferences, an intuïtive formal semantics is further proposed by Britz et al. (2011) [13]. In effect, the semantics of preferential DLs is organized in terms of strictly partially-ordered structures, \( \mathcal{P} := (\Delta^P, \preceq^P, \prec^P) \), where:

\[
\begin{align*}
- \quad (\Delta^P, \preceq^P) & \text{ is an ordinary DL interpretation;} \\
- \quad \prec^P & \text{ is an irreflexive, anti-symmetric and transitive partial order on } \Delta^P; \\
- \quad \text{ and } \prec^P & \text{ is, additionally, smooth: for every } C \in \mathcal{L}, \text{ if } C^P \neq \emptyset, \text{ then } \min_{\prec^P} (C^P) \neq \emptyset, \text{ where } \min_{\prec^P} \text{ denotes the minimal elements in } C^P.
\end{align*}
\]

The partial order organizes the elements of an arbitrary domain \( \Delta^P \) in a stratification corresponding to several “levels of typicality”, decreasing from bottom to up; that is, the lower-level objects must be the most normal. The set of allowed stratifications is informed by the knowledge specified in the knowledge base in the form of defeasible concept inclusions.\(^3\) Given a preferential DL interpretation \( \mathcal{P} \) and a defeasible subsumption statement \( C \sqsubseteq D \), the semantics of the latter is given by:

\[
\mathcal{P} \models C \sqsubseteq D \text{ iff } \min_{\prec^P} (C^P) \subseteq D^P.
\]

\(^3\) We can make an analogy with what happens when specifying propositional knowledge bases. Under an empty knowledge base, all possible propositional valuations are allowed; then if one adds \( p \land q \) to the base, only the valuations satisfying \( p \land q \) remain; and so on as more sentences are added. Here, in the preferential case, at the starting point we have all possible preferential interpretations, with all possible orderings on the domain. After adding the statements \( C \sqsubseteq D \) and \( C \nsubseteq D \), the knowledge base, only those preferential interpretations which stratify the extension of \( C \) into the more typical ones (those that are not \( C \) and that fall under \( D \)) and the less typical ones (those that are also \( C \) and are not \( D \)) remain. The process then continues as more knowledge is added to the ontology and in the end only the orderings that are compatible with the knowledge base are the allowed ones.
From the rational and preferential subsumption definitions, we can proceed with the associated entailment definitions [13].

**Definition 2.5.** (Preferential Entailment). A defeasible subsumption statement $C ▼ D$ is preferentially entailed by a given defeasible knowledge base $KB$ iff $C ▼ D$ is a statement of the preferential closure of $KB$, i.e., it is a derivation from $KB$ using the rules of Preferential Subsumption.

In such context, nevertheless, given a preferential interpretation $P$, we cannot conclude (in general) $P ⊨ C ▼ D$ from $P ⊨ C ▼ D$. Britz et al. (2011) [13] mention that “preferential entailment is thus too weak”. Inferences drawn in the legal context should be retractable and ampliative [23]. In other words, plausible (though provisional) conclusions need to be inferred in the absence of conflicting information. According to it, such conclusions should be removed in the light of new conflicting information. Therefore, we leave aside the monotonic preferential entailment, and adopt here the non-monotonic rational entailment [25].

Rational entailment solves the unwanted property of monotonicity induced by preferential interpretations through a preference ordering on models [25]. The motivation for this is to realize that some models are more important than other.

**Definition 2.6.** (Rational Entailment). A defeasible subsumption statement $C ▼ D$ is rationally entailed by a given defeasible knowledge base $KB$ iff $C ▼ D$ is a statement of the rational closure of $KB$, i.e., it is a derivation from the minimal model (the model in which objects are as typical as possible) of $KB$.

In the following example, we address these particularities.

**Example 2.1.** Suppose a knowledge base $KB_{bf}$ conceptualizing the domain of birds. We have three concepts: Bird, Penguin, and Fly (representing entities with flying capabilities, like birds, mammals, airplanes). For $KB_{bf}$, terminological axioms define classical and defeasible subsumption statements, while the Abox asserts four famous cartoon instances, namely happyFeet, skipper, woody and bartok.

$$KB_{bf} = T \cup A = \{\text{Fly}(\text{skipper}),\ \text{Fly}(\text{woody})\}$$

By axiomatizing the knowledge of flying objects, happyFeet was asserted as a Penguin. Nothing very specific was said about skipper and woody except that they are birds. Under the principle of presumption of typicality [26], it is plausible to infer that these individuals do fly. It is assumed, therefore, that they are typical instances of birds. Nevertheless, taking into account that we are dealing with partially observable environments, new acquired information may require the withdrawal of some previously inferred consequences. This is what happens upon discovering that skipper is a penguin. In these terms, Fly(skipper) needs to be retracted from the set of inferences produced.\(^4\)

Figure 1 illustrates a possible interpretation ($P$) satisfying the new knowledge base $KB'_{bf}$. $P$ is defined in terms of $(\Delta^P, P, \prec^P)$, where:

$$\Delta^P = \{\text{happyFeet}, \text{skipper}, \text{woody}, \text{bartok}\};$$

$$P^P = \{\text{happyFeet}, \text{skipper}, \text{woody}\};$$

$$P^P = \{\text{woody}, \text{bartok}\};$$

$$\prec^P = \{(\text{woody}, \text{happyFeet}), (\text{woody}, \text{skipper}), (\text{bartok}, \text{happyFeet}), (\text{bartok}, \text{skipper})\}.$$  

$$KB'_{bf} = \{T = \{\text{Bird} \sqsubseteq \text{Fly}, \text{Penguin} \sqsubseteq \text{Bird}, \text{Penguin} \sqsubseteq \neg \text{Fly}\}, \ A = \{\text{Penguin}(\text{happyFeet}), \text{Penguin}(\text{skipper}), \text{Bird}(\text{skipper}), \text{Bird}(\text{woody}), \text{Fly}(\text{bartok})\}\}$$

In the class of birds, woody is the minimal object with respect to penguins happyFeet and skipper. Even outside the set of birds, so does bartok in relation to the penguins. It is assumed that the most typical elements of Bird and Fly indeed fly. Remaining objects (arranged at the highest level) are exceptions to these, i.e., cases

\(^4\)The principle of presumption of typicality is one of the fundamental principles of rationality in non-monotonic reasoning. It is at the heart of a form of ampliative reasoning and states that we shall always assume that we are dealing with the most typical possible situation compatible with the information at our disposal.  

\(^5\)It is worth pointing out here that rational entailment check (i.e., the computation of rational closure) has to be recalculated. Our use of retraction here refers to what happens at the level of all logical consequences of a base: from the user’s perspective, it is as if a piece of knowledge had been retracted.
where generalizations do not apply. If, instead, we claim that Fly subsumes Bird in the traditional sense, that is, Bird $\sqsubseteq$ Fly, the base would be inconsistent: $KB\text{}_b\text{f}| = \top\sqsubset\bot$, since $KB\text{}_b\text{f}| = Penguin \sqsubseteq \bot$ (from $KB\text{}_b\text{f}| = Penguin \sqsubseteq Fly$ and $KB\text{}_b\text{f}| = Penguin \sqsubseteq \neg Fly$) and Penguin(happyFeet) $\in KB\text{}_b\text{f}$.

We end the present section with a discussion on the advantages of preferential description logics over their classical counterparts and on some philosophical and technical reasons for employing them rather than their classical cousins.

One may object that exceptions and overruling of properties can be formalized in classical DLs by fully specifying in the left-hand side of subsumption statements all the conditions for a rule to hold. For example, one could state Bird $\sqcap \neg Penguin \sqsubseteq Fly$ and Penguin $\sqsubseteq \neg Fly$. As it turns out, from a knowledge representation and reasoning perspective, this is not suitable. The reasons are as follows:

- The first reason relates to readability of the statements in the knowledge base. The more conditions one explicitly adds to the left-hand side of subsumption statements, the longer and harder to understand such statements become.
- The second reason is that every discovery of a new exception will require remodelling of the whole knowledge base, which is a computationally expensive task, even when performed offline.
- The third reason relates to the way we humans actually reason under incomplete information. When performing reasoning, we humans do not explicitly think of all special cases that would prevent a conclusion from being drawn. Instead, we base our reasoning only on the information at our disposal and provisionally jump to the conclusion. It is only upon facing new information that we accommodate it with the previous knowledge we had and, usually, we do it in a non-disruptive way.
- The fourth reason is that classical DL entailment is non-ampliative and non-defeasible, features that are sought for when reasoning with incomplete information.

We contend the preferential approach to default reasoning addresses the four points above in a satisfactory manner:

- First, our defeasible concept inclusions are shorter than the classical ones explicitly specifying the exceptions and therefore are user-friendly.
- Second, since the statements in the knowledge base already cope with exceptions, the discovery of a new one is automatically dealt with. Remodelling is only required if really desired, e.g., if one wants to refine the levels of typicality, and even in this case changes to the knowledge base are localized.
- Third, the reading of a defeasible statement of the form $C \sqsubseteq \neg D$, i.e., “usually, Cs are Ds” is in line with human actual reasoning.
- Finally, rational closure is a well-accepted form of ampliative and defeasible entailment relation.

3. Nuances of the Legal Domain

Throughout this section, we argue on the limitations of classical DL to tackle a kind of normative conflict inherent to the legal domain, in particular a type often observed within the normative corpus of criminal law.

3.1. Normative Conflicts and the Double Jeopardy Principle

According to Lindahl (1992) [8], a normative conflict occurs in a situation where it is impossible to apply two rules together. Boer et al. (2005) [9] list the three main types of normative conflicts: Disaffirmation conflict, Compliance conflict, and conflicts between rules that are defeasible. Disaffirmation refers to situations where an event is qualified by two different deontic mode norms (permissive and prohibitive, for example). The conflict known as compliance refers to situations where norms with the same deontic qualifica-
 tion still remain opposed. Finally, the last conflict deals with normative rules that are defeasible, to prevent, for example, situations in which one ought to do substantially the same thing twice in different ways. The latter is the type of conflict addressed in this manuscript.

Usually, legal systems explicitly define a preference among norms, which are hierarchically ordered. From more general and comprehensive norms, such as the sovereign constitution, other laws, decrees and regulations are derived. In legal theory [27], usually when two laws govern the same facts, a law governing a specific subject matter overrides a law governing only general matters. As a result, legal reasoning is defeasible by nature. Inferred conclusions can potentially be withdrawn, with the inclusion of new information that triggers more specific situations. Celano (2012) [28] argues that, although seen as conditional rules of type IF <CONDITIONS> THEN <CONSEQUENT>, the antecedent part of a norm must be satisfied only under normal conditions. This is because the application of legal rules are open to be canceled by specific circumstances (which are either hidden within the lines of normative texts, or are clearly explained by them). In general, however, those situations are difficult to specify prior to the application (ex ante) of the law for particular cases [23]. Worse yet, it is humanly impossible to list all circumstances and sufficient conditions for practical applications [29]. Figures 2 and 3 depict a situation represented in classical and preferential DL, along with inferences of (in)consistency.

Within the Penal Code, ordinary crimes are built from more typical situations (written down in the caput of articles), and other criminal types (more serious or soft) unfold from this. In the criminal realm, for example, one cannot condemn the behavior of an agent such as theft (art. 155) and robbery (art. 157) at the same time. In Figure 2, using classical DL, events of stealing match a theft; these events accompanied by aggression, characterize a robbery. Besides, each crime defines its unique sanction. On these conditions, it is inferred that Robbery is a Theft. However, from this new knowledge, with those already laid down at the knowledge base, two penalties are imposed on the crime of robbery, leading to an inconsistency, since only one penalty shall be prescribed to a crime. Although not explicitly illustrated, we assume that the inconsistency is reached by TBox together with ABox; this latter is represented by e1 and e2 individuals. In this case, e2 should be classified as Theft and Robbery, and therefore, relating the instance to both penalties.

Nonetheless, in Figure 3, there is no longer any inconsistency in the imposition of penalties, due to the use of Preferential DL. According to the Preferential DL semantics, individuals can be ordered by typicality inside a class; therefore, only the most typical stealing events are classified as theft. Robbery events would be at a level above. Note that this is due to the use of the preferential subsumption relation, contrary to the classical, monotonic DL semantics. Such an example shows why preferential approaches seem to fit more naturally to law subsumption in general, and helps building law hierarchies where, despite specific laws are viewed as special cases of more general laws, the penalty conflict is dealt with the usual way law practitioners do. The axioms will be explained in the following sections.

It is still worth mentioning that our objective is to help in classifying the different types of acts, with the consequent imposition of penalties. Certain idiosyncrasies resulting from classical reasoning are thus avoided.

3.2. A Case Study about Crimes Against Property

The Brazilian Criminal Law is referred to as the set of legal rules that define criminal offenses, establishing punishment and security measures. These norms establish definitions about the crimes, their types, and criminal penalties. Besides, they are structured by the Brazilian Penal Code7 (Decree-Law 2.848 dated 1940), having been extensively amended by Law 7209/1984. Notably, crimes against property correspond to the protected legal interest in the crimes set out in Articles 155-180. Beyond the economic value, for criminal purposes, the value of assets covers also the moral value of goods, as a letter, a stone, or any material object that has affection value to the owner, although it may not have an exchange value. Crimes against property include, but are not limited to (the original norms in native Portuguese are available in Appendix A):

- **Theft**: To take away a chattel8, for himself or others. (Art. 155);
- **Robbery**: To take a chattel, for himself or others, by serious threat or violence [...] (Art. 157);

---

7http://www.planalto.gov.br/ccivil_03/decreto-lei/del2848.htm
8An item of personal property that is movable.
Robbery followed by Death: If the violence results in [...] death, the jail time is twenty to thirty years. (Art. 157, § 3º);
Example 3.1. Bill is a graduate student and attends classes at night shift. One day, returning home, Bill was approached by a biker who ordered Bill to give him his wallet. The biker was armed and said that if Bill told anyone on the wallet, he would kill him.

For legibility, criminal domain concepts are structured in terms of Unified Modelling Language (UML) classes [30]. Figure 4 pictures a class diagram for a partial view of the General Theory of Crime, as stated in the Brazilian Penal Code. 9 Herein, we have specialized the top ontology known as Unified Foundations Ontology (UFO), which deals with the duality of established philosophical theory between Endurant and Perdurant entities [32, 33]. Endurants are perennial particulars that exist as time goes by (like agents and objects). Situations are special sorts of endurants as well, a portion of reality recognized as a whole, a state of affairs. Conversely, Perdurants happen in time, that is, they are framed by intervals of time. Substantial instantiate a Kind category, i.e., a rigid type which provides the essential property (that is, any substantial instantiate imperiously in every situation). UFO provides further non-rigid categories, such as Role, which is instantiated only in certain contexts depending on a given relation, whereas Phase is a type instantiated in a context dependent on an intrinsic property.

Unlike (non-agentive) objects, herein we consider Agent as any entity that can initiate an Action. CrimeAgent is a top category for any Agent within the criminal realm. It specializes further in two disjoint roles, agents who perform a criminal act (ActiveAgent) and agents who have had some property protected by the State (honor, life, valuable object, and so on) violated (PassiveAgent). Situations are changed through a CriminalAct, that is, an Action, carried out by the ActiveAgent. CriminalAct violates a CrimeObject, which is in turn, associated to a PassiveAgent.

A ForbiddenSituation refers to a specific state-of-affairs prohibited by an IncriminatingRule. These situations are composed by violated crime objects, and injured passive agents. The kernel of a Crime is the criminal action taken. As secondary elements, a legal act may have additional (aggravating/mitigating) circumstances, modifying the crime type. In order not to overload the figure, we avoid illustrating details of norms and sanctions. Figure 4 pictures the axiomatization for the theory of crime.

9In [31], we have provided a detailed axiomatization with respect to the Theory of Crime.

Figure 5 illustrates the instantiation for Example 3.1. behaviorBob is a complex event involving two actions, carried out by the same agent. Each violates a legal asset associated with the passive agent. The resulting situation is forbidden by the article that typifies the robbery, but also by the theft article (when considering only part of the final situation).

The TBox $\mathcal{T}$ models both types of crime. Conversely, the ABox $\mathcal{A}$ models Bob’s behavior.
between classes in the UML class diagram. We also avoid overloading ABox $\mathcal{A}$ with too many axioms. Among other results of logical subsumption, classical inference reasoning in DL shows further that (given $\mathcal{KB} = \mathcal{T} \cup \mathcal{A}$):

$$\mathcal{KB} \models \{ \text{Theft}(\text{behaviorBob}), \text{Robbery}(\text{behaviorBob}) \}$$

From a logical point of view, the axiomatization in Example 3.1 is consistent. According to what has been stated, Bob is classified as an $\text{ActiveAgent}$ instance. More importantly, his behavior is classified in two ways. The consequence entailed by $\mathcal{KB}$ classifies $\text{behaviorBob}$ as both a theft and a robbery. Although we have not shown in Figure 4, a criminal norm defines the sanctions associated. Instance $\text{art155Law2848}$ defines as punishment a prison time between 1 and 4 years, while instance $\text{art157Law2848}$ defines a prison time between 4 and 10 years.
There are some slight modifications in the axioms (1) and (2). The latter axioms state that, typically, stealing something is a theft (1). New conditions may defeat this conclusion: stealing accompanied by violence or verbal threat is a robbery (2). Nevertheless, under the semantics of rational consequence (particularly, regarding the rational monotony property), given the premises:

\[
\begin{align*}
\text{Event } \not\exists \text{realizedThrough. Steal} \subseteq \text{Theft}, \\
\text{Event } \not\exists \text{realizedThrough. Steal} \subseteq \neg \text{realizedThrough. Aggression},
\end{align*}
\]

we still can infer:

\[
\begin{align*}
\text{Event } \not\exists \text{realizedThrough. Steal} \exists \text{realizedThrough. Aggression} \subseteq \text{Theft}
\end{align*}
\]

Based on this conclusion, even taking into account the new knowledge base \( KB_d = T_d \cup A \), as we know that Robbery \( \subseteq \neg \text{Theft} \), then:

\[
KB_d \models \bot
\]

The knowledge base \( \langle A, T_d \rangle \) still does not match the expected practical result in the legal field, since dual classification violates the principle already mentioned and makes the knowledge base inconsistent. The RM property still forces the inference of theft. It is necessary to maximize the typicality of objects; in the context, to inform that, normally, events carried out through stealing are not accompanied by aggression:

\[
T'_d = T_d \cup \{ \text{Event } \exists \text{realizedThrough. Steal} \subseteq \neg \text{realizedThrough. Aggression} \}
\]

Thus, from \( KB_d' = T'_d \cup A \), we reach the desired conclusion:

\[
KB_d' = \text{Robbery(behaviorBob)}, \\
KB_d' \nvDash \text{Theft(behaviorBob)}
\]

Based on the notions of typicality and preference aforementioned, in the next subsection we propose a preferential and rational overrule relations to hierarchically organize a set of axioms within an arbitrary legal knowledge base. Some mathematical properties of these relations are given as well.
4.1. Preferential DL for Double Jeopardy Principle

The non-monotonic subsumption relation used in this study establishes an implicit priority among defeasible axioms. Given that, the contribution of the present paper is the provision of an approach for modelling legal domains in the presence of the Double Jeopardy principle. In other words, we propose a non-monotonic specification of crimes of the criminal code, based on an implicit ordering of precedence on norms. For this, we define overrule relations and denote when they are (implicitly) applied under preferential and rational semantics. In effect, we have addressed the Priority of Specificity (according to Béltran and Ratti 2013 [23]), which presupposes that more specific information cancels out more general ones. Consider that for each defeasible subsumption axiom γ, it has an antecedent (A) and a consequent (C) part, expressed as follows:

\[ \gamma : A(\gamma) \sqsubseteq C(\gamma) \]

From this, we propose the following overrule relations.

**Definition 4.1. Preferential Overrule Relation \((\succ_p)\).** Let us take \( \sqsubseteq \) as a preferential subsumption relation (Definition 2.3). Thus, given a consistent defeasible TBox \( T \) and two defeasible axioms (α and β), in which \( T \models \alpha \) and \( T \models \beta \), and neither \( T \models C(\alpha) \sqsubseteq \bot \) nor \( T \models C(\beta) \sqsubseteq \bot \). We say β overrules α (under the preferential semantics) if and only if \( T \models A(\beta) \sqsubseteq A(\alpha) \), and \( T \models C(\beta) \sqsubseteq C(\alpha) \sqsubseteq \bot \). Thus, in general, \( \beta \succ_p \alpha \) if:

\[
T \models \left\{ \begin{array}{l} \alpha \models \bigwedge_{i=1}^{m} D_i \sqsubseteq C(\alpha) \\ \beta \models \bigwedge_{i=1}^{n} D_i \sqsubseteq C(\beta) \\ \eta : C(\alpha) \sqcap C(\beta) \sqsubseteq \bot \end{array} \right. 
\]

By handling the exception between α and β, it must be stated that their consequent portions are disjoint (η axiom). Otherwise, an individual could be classified as both concept expressions (\( C(\alpha) \) and \( C(\beta) \)); which would not lead to an exception.

**Lemma 1.** The preferential overrule relation \( \succ_p \) is irreflexive.

**Proof.** Assume that \( \succ_p \) is reflexive. Thus, given a defeasible axiom \( \alpha \) and a TBox \( T \), where \( T \models \alpha \), then \( \alpha \succ_p \alpha \). This leads to a contradiction, with Definition 4.1, since \( T \not\models C(\alpha) \sqcap C(\alpha) \sqsubseteq \bot \), taking into account the hypothesis that \( T \not\models C(\alpha) \sqsubseteq \bot \).

**Lemma 2.** The preferential overrule relation \( \succ_p \) is asymmetric.

**Proof.** Assume that \( \succ_p \) is not asymmetric. Hence, there are at least two defeasible axioms \( \alpha \) and \( \beta \), and a TBox \( T \), where \( T \models \alpha \) and \( T \models \beta \), and \( \alpha \succ_p \beta \). From Definition 4.1, we have \( A(\beta) \sqsubseteq A(\alpha) \) and \( A(\alpha) \sqsubseteq A(\beta) \). In other words, \( A(\alpha) \equiv A(\beta) \). However, still according to Definition 4.1, the consequent part of the axioms are disjoint, and hence: \( T \models C(\alpha) \sqcap C(\beta) \sqsubseteq \bot \). Similarly to the last proof, assuming the hypotheses \( T \not\models C(\alpha) \sqsubseteq \bot \) and \( T \not\models C(\beta) \sqsubseteq \bot \), and as we already know that \( A(\alpha) \equiv A(\beta) \), thus \( A(\alpha) \sqsubseteq C(\alpha) \) and \( A(\alpha) \sqsubseteq C(\beta) \). Clearly we would have \( T \models T \sqsubseteq \bot \), since a typical instance of \( A(\alpha) \), by the preferential/rational subsumption, could not be simultaneously classified as disjoint concepts. In other words, from the previous axioms, we would have \( A(\alpha) \sqsubseteq C(\alpha) \sqcap C(\beta) \), thus \( A(\alpha) \sqsubseteq \bot \). Hence, we confirm the asymmetry of the overrule relation.

**Lemma 3.** The preferential overrule relation \( \succ_p \) is intransitive.

**Proof.** Consider the following \( KB \), with arbitrary concepts:

\[
KB \models \left\{ \begin{array}{l} \alpha : X \subseteq Y \\ \beta : X \cap Z \sqsubseteq H \\ \gamma : X \cap Z \cap O \sqsubseteq P \\ Y \sqcap H \sqsubseteq \bot \\ H \sqcap P \sqsubseteq \bot \end{array} \right. 
\]

From Definition 4.1, we have \( \beta \succ_p \alpha \) and \( \gamma \succ_p \beta \). Nevertheless, as \( KB \not\models Y \sqcap P \sqsubseteq \bot \), we can not infer \( \gamma \succ_p \alpha \). On the other hand, considering three or more defeasible axioms, if all its consequent parts are pairwise-disjoint (as long as the criterion concerning its antecedent parts is also met), the transitivity property will be preserved. For the latter example, just note that adding \( Y \sqcap P \sqsubseteq \bot \) in \( KB \) will make the axioms \( \alpha, \gamma \) match all the criteria of the overrule relation.

**Lemma 4.** The preferential overrule relation \( \succ_p \) is acyclic.

From Lemma 2, it is trivially proven that there are no cycles between overrule relations. Once again, the veracity of the Lemma is assured only if the hypothesis that the consequent parts of the axioms are satisfiable. Therefore, if the cyclic property were true, for two axioms \( \alpha \) and \( \beta \), if \( \beta \succ_p \alpha \) then \( \alpha \succ_p \beta \). But this would
make the overrule relation also symmetrical, which has already been proven to be false in Lemma 2.

We provide the overrule relation for the rational subsumption relation as well. In order to facilitate understanding, we define an operator of difference between DL formulae.

**Definition 4.2. Specificity Subtraction.** Given two DL concepts \( \mathcal{F}_1 \) and \( \mathcal{F}_2 \), where \( \mathcal{F}_1 \subseteq \mathcal{F}_2 \) of the form \( \bigcap_{i=1}^{m} E_i \), \( \alpha_i \) is an operator of specificity subtraction, where \( \mathcal{F}_1 \setminus \mathcal{F}_2 \) results in a DL formula \( \mathcal{F}_3 \), where \( \mathcal{F}_2 \cap \mathcal{F}_3 \equiv \mathcal{F}_1 \).

**Definition 4.3. Rational Overrule Relation \( (\triangleright) \).** Let us take \( \sqsubseteq \) as a rational subsumption relation (Definition 2.4). Thus, given a defeasible TBox \( T \) and two defeasible axioms \( \alpha \) and \( \beta \), such that \( T \not\models \alpha \) and \( T \not\models \beta \), and neither \( T \models C(\alpha) \equiv \bot \) nor \( T \models C(\beta) \equiv \bot \). We say \( \beta \) overrules \( \alpha \) if and only if \( T \models A(\beta) \sqsubseteq A(\alpha) \), \( T \models C(\beta) \sqsubseteq C(\alpha) \), and \( T \models A(\alpha) \sqsubseteq \neg F \), where \( F = A(\beta) \setminus DL A(\alpha) \). Thus, in general, \( \beta \triangleright \alpha \), if:

\[
\mathcal{T} \models \left\{ \begin{array}{l}
\alpha : \bigcap_{i=1}^{m} D_i \sqsubseteq C(\alpha) \\
\beta : \bigcap_{i=1}^{m} D_i \setminus \bigcap_{j=1}^{n} E_j \sqsubseteq C(\beta) \\
\gamma : C(\alpha) \setminus C(\beta) \sqsubseteq \bot \\
\delta : \bigcap_{i=1}^{m} D_i \setminus \neg (\bigcap_{j=1}^{n} E_j) \sqsubseteq \neg F \end{array} \right. \\
\mathcal{T}_\alpha = \left(\begin{array}{c}
\mathcal{T} = \left\{ \begin{array}{l}
\alpha : \bigcap_{i=1}^{m} D_i \sqsubseteq C(\alpha) \\
\beta : \bigcap_{i=1}^{m} D_i \setminus \bigcap_{j=1}^{n} E_j \sqsubseteq C(\beta) \\
\gamma : C(\alpha) \setminus C(\beta) \sqsubseteq \bot \\
\delta : \bigcap_{i=1}^{m} D_i \setminus \neg (\bigcap_{j=1}^{n} E_j) \sqsubseteq \neg F 
\end{array} \right. \\
\mathcal{T}_\alpha = \left(\begin{array}{c}
\end{array}\right)
\end{array}\right) \\
\end{array}
\]

and \( \bigcap_{j=1}^{n} E_j \equiv A(\beta) \setminus DL A(\alpha) \).

Lemmas 1-4 apply directly to rational overrule relation.

The next section details a case study within the Brazilian criminal code governing crimes against property.

### 4.2. Expanding Specificity Levels in Crimes against Property

In this section, we continue with the case study of crimes against property, leveraging the previous example with new additional circumstances. A new type of crime is introduced: Robbery Followed By Death. Henceforth, we use Example 4.1 to illustrate the levels of specificity between norms.

**Example 4.1.** Bill is a graduate student and attends classes at night shift. One day, returning home, Bill was approached by a biker who ordered Bill to give him his wallet. Frightened, Bill tried to run, but Bob shot him to death. Bob picked up Bill’s wallet lying on the floor, and walked away.

We present the knowledge base for the subset of crimes against property \((KB_p)\). Only for didactic reasons, we separate the Tbox into two distinct sets, splitting the classical and defeasible axioms (respectively, \( T_c \) and \( T_d \)). Additionally, the Example 4.1 is described shortly after by the corresponding assertional axioms. Figure 6 models the corresponding individuals.

This object diagram was also instantiated based on the classes and associations highlighted in Figure 4.

We emphasize the concept of Deceased, that is, that agent who had his life (a crime object) violated by some criminal action.
Thus, we can easily verify the following rational overrule relations (definition 4.3) between the defeasible axioms:

$$\text{EventOfSteal} \land \text{EventOfAggression} \supset \exists \text{hasPosState}. (\exists \text{hasEndurant}. (\exists \text{PassiveAgent}. (\exists \text{Deceased}. (\text{Theft} \land \text{RobberyAndMurder} \land \text{behaviorBob} \supset \text{EventOfSteal} \subset \text{RobberyAndMurder} > \text{Theft} \circ \text{behaviorBob} \circ \text{RobberyAndMurder})))$$

Additionally, considering the Example 4.1, we have: $KB_{w} \models \text{RobberyAndMurder}(\text{behaviorBob})$ and $KB_{w} \not\models \text{Theft}(\text{behaviorBob})$ and $KB_{w} \not\models \text{Robbery}(\text{behaviorBob})$.

### 4.3. Reasoning with the DIP Protégé Plug-in

Given other numerous DL extensions with defeasibility (detailed in Section 5), as far as we know, few robust and scalable implementations have been made available. With respect to the Preferential DL, an existing Protégé plugin can be used, at least at the TBox level, for defeasible reasoning under the rational semantics, known as DIP (acronym for Defeasible Inference Platform) [34].

DIP is composed of three view windows: one for regular axioms (Figure 7 (1)), one for defeasible axioms (Figure 7 (2)), and the DIP view itself for reasoning (Figure 7 (3)). In order to tag an axiom as defeasible, a toggle button (d) is available. Another point to consider is that DIP makes call to a classical reasoner.
FACT++. Figure 7 illustrates these windows within DIP tab.

For the illustrated scenario in Figure 7, the query describes a situation where there is a dead agent (the victim). Subsequently, two actions were taken to arrive at this state-of-affairs, a theft of an item, and an aggression. In addition to the rigid superclasses, DIP ranked the situation as only a crime of RobberyAndMurder.

More importantly, no other crime has been mentioned, although the query involves the inherent circumstances of the robbery and theft crimes. Figure 8 illustrates the situation for typical cases of robbery.

As a result of described actions, the dead person is a crucial item to meet the exceptionality criteria. The absence of this particularity classifies the situation as a crime of Robbery, as depicted in Figure 8. Even so, the presence of the steal event does not inflict the behavior as a simple crime of Theft.

The fact that Preferential DL can be reasoned automatically within an ontology environment certainly consists in one more advantage for Preferential DL, a feature that most of the other non-monotonic alternatives cannot offer.

5. Related Work

In this section, we highlight similar studies to deal with non-monotonic nuances in the legal domain, and alternative proposals to accommodate typical and non-typical cases in DL formalism. In the end, we present a comparative table between these last studies, singling out why we chose the preferential approach.

5.1. Handling Conflicts in the Legal Domain

Since the 80s, debates about the need of non-monotonicity as a necessary requirement for legal reasoning have been conveniently carried out, even persisting to this day [35]. Situations such as the judicial reasoning over incomplete (usually, unknown information), the open-textured concepts [36], the heterogeneity of legal sources, and the very interplay among laws with legal principles (which operate on a higher level of reasoning w.r.t. laws, closing some gaps left by them) [37] are potentially sources of conflicts.

In addition to the preferential approach, non-monotonic logics have been widely used to deal with conflicts. Some of these approaches are: Reiter’s default reason-ing [38], McCarthy’s circumscription approach [39] and the logical theory for defeasible reasoning of [40]. Other studies focused on approaches that set order of preference between elements in conflicts, either in a rule-based [41], [42], or on legal arguments perspective [43], [44], [45]. It is also important to note that some papers [46], [47] have tried to define levels of reasoning, where monotonic and non-monotonic logic could coexist harmoniously. For such solutions, at lower levels, from the current circumstances, facts (arguably conflicting) are inferred. At a higher level, if necessary, the problems would be solved. That is, goal-setting rules at a second level would try to set priorities (often presupposed by lawyers and judges), and validate rules at a lower level.

5.2. Proposals for Handling Exceptions in DL

There are already studies about the problem of dealing with exceptions in ontologies, even inside the Description Logic community. Let us consider a sketch of five different approaches, in order to compare them to our strategy:

- Neo-topological approach to reasoning on ontologies with exceptions proposed by [48];
- Defeasible description logics based on (defeasible) rules proposed by [49];
- A Non-monotonic description logic $ALC+T_{min}$ which defines an operator of typicality for description logic under a preferential semantics, proposed by [50];
- A circumscriptive approach proposed by [51], and finally
- A new semantics designed to address knowledge engineering needs (called $DL^N$), proposed by [52].

“Neo-Topology” regards entities as points in space, classes as clusters or groups of entities in space, and classical topological operators (interior, border, and closure). Additionally, neo-topology also uses the concept of Typicality Degree to assign an element “L” (in a class) which does not match all the properties of a class, and creates the notion of “thickness” within the border. This defines a topological area where it is possible to assign those elements of the class which are neither typical nor atypical, but rather falling in

11In this subsection, as we are not specifically addressing the legal domain, rather than conflicts, we speak of exceptions in a general way.
between. Doing so, neo-topology might express exceptions and even exceptions of exceptions. Although neo-topology has a user-friendly graphical representation, relationships between elements are not defined, different from description logic formalisms. Besides that, as far as we know, there are no results about complexity of neo-topology reasoning at all. In addition, the solution fails to have a clear underlying formalism, and the rules of inference are limited to the operations of conjunctions.

A hybrid system to extend DL with defeasible rules is the proposal by [49]. Actually, the intention is to anchor DL in a Defeasible Logic, particularly the DefL (Defeasible Logic), proposed by [53]. Defeasible Logics are rooted in an underlying defeasible theory [40], [54], which predicts defeaters rules, and a superiority relation between (defeaters) rules. A defeasible the-
ory \( \mathcal{D} \) is a structure \( \mathcal{D} = (\mathcal{F}, \mathcal{R}, \succ) \) where \( \mathcal{F} \) is a finite set of grounded literals (the facts), \( \mathcal{R} \) is a finite set of FOL rules, and \( \succ \) is an acyclic binary relation of superiority on \( \mathcal{R} \). Thus \( \alpha \succ \beta \) states that \( \alpha \) overrules \( \beta \) if both rules are applicable, and these rules have complementary head literals. DEF-\( ALC \) is the result of a translation (\( \tau \)) applied to the ABox/ TBox of a DL base in a language of a defeasible theory. In particular, the attempt to integrate the systems is only achieved through a new non-monotonic operator: \( \subseteq \), in order to provide an operator isomorphic to the DefL defeasible rules. Given \( \mathcal{C}p1, \mathcal{C}p2 \) as concepts, \( \rightarrow \) and \( \Rightarrow \) as the DefL classical strictly implication and defeasible implication (respectively), we have:

\[
\begin{align*}
\tau(\mathcal{C}p1 \subseteq \mathcal{C}p2) &= (\tau(\mathcal{C}p1) \rightarrow \tau(\mathcal{C}p2)) \\
\tau(\mathcal{C}p1 \nsubseteq \mathcal{C}p2) &= (\tau(\mathcal{C}p1) \Rightarrow \tau(\mathcal{C}p2))
\end{align*}
\]

Still, as expected, the proposal provides a binary superiority relation (\( \succ \)) between the defeasible rules. The solution presented has several drawbacks, however. The translation does not address the blocking rules of the form \( \mathcal{C}p1 \sim \sim \succ \mathcal{C}p2 \) [54]. Furthermore, no data were presented about the proposal complexity, besides the overwork in translation between formalisms. Another point is the user’s need to make explicit new rules, whenever new exceptions to superiority relations have emerged. Something that is often not a trivial task.

The DL extension proposed by [50], presents a semantics based on the KLM preferential semantics for non-monotonic reasoning. \( ALC + \text{T}_{\text{min}} \) contains a new operator (\( \text{T} \)) of which the intuition is to single out the typical instances, rather than to extend some regular DL operator. As expected, operator \( \text{T} \) preserves the postulates stated by KLM. In order to leverage the power of non-monotonic reasoning at its maximum, \( ALC + \text{T}_{\text{min}} \) strengthens the semantics of monotonic \( ALC + \text{T} \) considering only the minimal model semantics. Therefore, given an arbitrary knowledge base \( \mathcal{K}B \), and \( \llbracket \mathcal{K}B \rrbracket \) as the set of models under \( ALC + \text{T} \), \( ALC + \text{T}_{\text{min}} \) maximizes typicality selecting the models from \( \llbracket \mathcal{K}B \rrbracket \) with the lowest number of atypical instances. By axiomatizing, for example, that, typically, stealing something from someone is a theft, is represented as follows:

\[
\text{T}(\text{EventOfSteal}) \subseteq \text{Theft}.
\]

Despite the well-founded semantics and the available tableau calculus decision procedure for checking minimal entailment, as far as we know, there is no robust implementation for \( ALC + \text{T}_{\text{min}} \); conversely, tooling is crucial to engineer domain and assert reasoning capabilities.

[51] introduce a circumscripive extension of \( ALC \). The idea behind the proposal is to augment the knowledge base with “abnormality predicates” (\( \text{Ab}_{p} \)), of which the extension is meant to be minimized in the reasoning process. In order to represent the aforementioned theft example, we define one of the following equivalent axioms:

\[
\begin{align*}
\text{EventOfSteal} &\nsubseteq \neg \text{Ab}_{\text{EventOfSteal}} \subseteq \text{Theft} \\
\text{EventOfSteal} &\subseteq \text{Theft} \cup \text{Ab}_{\text{EventOfSteal}}
\end{align*}
\]

Similar to [50]’s approach, the goal is to restrict inferences to models in which the extension of abnormality predicates is as minimal as possible. Circumscribed knowledge bases are defined in terms of a tuple \( (\sim, M, \text{Fix}, V) \), where \( M \) are the abnormality predicates to be minimized, \( \text{Fix} \) comprises the unchanged predicates, \( V \) corresponds to the predicates that vary, and \( \prec \) is a strict partial order over \( M \). Suppose, for example, the following knowledge base:

\[
\mathcal{K}B : \left\{ \begin{array}{ll}
\text{EventOfSteal} \nsubseteq \neg \text{Ab}_{\text{EventOfSteal}} \\
\text{EventOfSteal} \subseteq \text{Theft} \\
\text{Ab}_{\text{Fix}} \cdot \text{EventOfAggression} \nsubseteq \text{Robbery} \\
\text{EventOfAggression}(\text{behavior}X), \text{EventOfSteal}(\text{behavior}X)
\end{array} \right\}
\]

For correct inferences about \( \text{behavior}X \), it is imperative to make explicit the priorities among the abnormality predicates. Therefore, the following axiom is added to the base, to infer that \( \text{behavior}X \) is a Robbery:

\[
\text{Ab}_{\text{EventOfSteal}(\text{behavior}X)} \cdot \text{EventOfAggression} \prec \text{Ab}_{\text{EventOfSteal}(\text{behavior}X)}.
\]

Deciding priority relationships adds an extra workload in ontological engineering. Besides, mistaken decisions can lead to counter-intuitive inferences. Still, the worst problem of this solution is the computational complexity with respect to the underlying entailment relation: \( \text{NEXP}^{\text{NP}} \)-complete [55].

A new non-monotonic description logic named \( \mathcal{D}C^{N} \) was proposed by [52]. The motivations for the new formalism revolved around two purposes: to address issues related to the computational complexity of non-monotonic reasoning, and to solve shortcomings of prior non-monotonicity strategies, such as inheritance blocking, that is, exceptional concepts do not directly inherit the default properties of their superclasses. \( \mathcal{D}C^{N} \) handles conflicts within a knowledge
base through a new approach: unresolved conflicts are
evidence of missing knowledge, thus knowledge engi-
neers should use them as support for knowledge base
scaling and validation. Another point to be highlighted
is that \( \mathcal{DL}^N \) axioms can be converted to classical DL
axioms in polynomial time.

\( \mathcal{DL}^N \) address non-monotonicity through two con-
structs: Normality Concepts and Defeasible Inclusions
(DIs). The former refers to the standard instances of a
concept \( C \). That is, for each \( \mathcal{DL} \) concept \( C \), there is a
new concept name \( NC \). DIs are expression of the form
\( C \sqsubseteq_D D \) which means “by default, standard instances
satisfy \( C \sqsubseteq D \), unless stated otherwise”. In the latter
case, \( C \sqsubseteq_D D \) is overridden by a higher priority DIs. A
\( \mathcal{DL}^N \) knowledge base is a disjoint union, \( KB = S \cup D \),
where \( S \) comprises a finite set of classical axioms (the
strong portion), and \( D \) is a finite set of DIs. The pro-
posed logic resolves conflicts between non-monotonic
axioms by a strict partial order (\( - \), which is usually
based on specificity notion.

Although the notion of “typical instances” is not ad-
dressed, \( \mathcal{DL}^N \) was constructed with postulates analo-
gous to those found in KLM theory. However, the first
version of the \( \mathcal{DL}^N \) did not completely satisfy its KLM
version of the LLE postulate. A correction was pro-
based by [56]. Moreover, given the encouraging results
of scalability tests [52] for bases with tens of thou-
sands of concepts, the approach, although very recent,
emerges as a future line of research to deal with ex-
ceptions in modelling legal-normative knowledge. It is
worth pointing out though that the approach by Bon-
atti et al. assumes an underlying DL language that is
much less expressive than \( \mathcal{SHOQ} \), which is the main
reason for the good scalability of their results. Here
we assume a more expressive description language,
which caters for a wider class of applications. More-
over, an advantage of preferential DLs as proposed by
Britz et al. and as used here over the approach by Bon-
atti et al. is the fact that preferential DLs satisfy all
the basic properties that are acknowledge as impor-
tant by the non-monotonic reasoning community. Bon-
atti et al.’s framework turns out to fail some of them,
one of the reasons being the fact their semantic con-
struction is not preferential. In moving down to a less
expressive language but keeping our preferential se-
mantics, we believe we can get performance results
comparable to those by Bonatti and colleagues while
preserving the preferential properties.

Table 2 summarizes a brief comparison between the
approaches described in this section. Preferential DL
brings together a set of conditions that guided us to de-
line it as the approach used in this research. Besides
being based on a solid theory for non-monotonicity
[24], practical implementations that formalize the de-
feasible reasoning were conceived [57]. In especial
DIP [34], which already implements the Rational se-
manics. It is important to point out that the computa-
tional complexity of the implementation is the same as
that of the monotonic entailment relation; i.e., it is an
\( \text{EXPTIME} \)-complete problem.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Comparing the Related Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>References</td>
<td>Computational Complexity of Entailment</td>
</tr>
<tr>
<td>Neo-Topological Logic</td>
<td>[48] Unknown</td>
</tr>
<tr>
<td>Defeasible DL</td>
<td>w/Declarable rules</td>
</tr>
<tr>
<td>( \mathcal{AC^LC+T_{min}} )</td>
<td>[50] coNEXP^p</td>
</tr>
<tr>
<td>Circumscription</td>
<td>[55] NEXP^p</td>
</tr>
<tr>
<td>Preferential DL</td>
<td>[57] ( \text{EXPTIME} )-complete</td>
</tr>
<tr>
<td>( \mathcal{DL}^N )</td>
<td>[52] ( \text{P#2ExpTime} )</td>
</tr>
</tbody>
</table>

6. Contributions and Final Remarks

The maturity of the Semantic Web technologies
has fostered the creation of more intelligent and au-
tonomous web services, capable of interacting with
one another and making decisions. Although not rela-
tively new, the legal area provides a challenging field
for knowledge conceptualization and reasoning, with
many possible semantic pitfalls. DLs have been widely
employed in the engineering of legal semantic mod-
els, with norms and regulations broken down into
manageable-sized ontologies. Nevertheless, little ex-
plored in the axiomatization of the legal domain are
the possibilities to overcome the limitations of classi-
cal DL, in particular, to handle conflicts that remain in
normative knowledge. Exceptions accommodate and
regulate the particularities of men living in society. Us-
ing them is not synonymous to error in the drafting of
laws, but rather, solutions so that juridical doctrine can
4 evolve along with social and political changes.
In this paper, we have made a case for a defeasible
5 extension of DL as a more suitable logical formalism for the nuances that are intrinsic to the legal domain. What was written in the Brazilian penal code as classic types of subsumption, in fact, represent special situations that, when not well conceptualized, lead to conclusions that contradict the doctrines of the juridical order. We have defined a theoretical foundation for the principle of specificity, where more specific laws subjugate other generic regulations, whenever their sanctions are likely to lead to inconsistency. In consequence, it preserves further the principle of Non bis in idem.

We have presented conceptualizations of some crimes from the Brazilian penal code through a slight modification in the implication operator of DL: $\sqsubseteq_L$. Such modification allows us to separate instances of a concept into two groups, the most typical, which obey the rules of modus-ponens, and those that represent special cases with respect to the base case. Therefore, inferences can be gradually dropped out, as much as new information is being gathered. As evidences and other legal artifacts can be set while trials are proceeding in court, it is of crucial importance to provide reasoning systems that draw conclusions which are no longer supported.

The applicability of this study emerges through Scientific-Technological and Legal contributions. The former addresses the advance of the use of DLs for the unambiguous and shared representation of knowledge, favoring transparency and effectiveness of law enforcement and the elimination of the gap between the use of technologies and legal systems. In the legal context, as we tighten the links between these areas, we foster a field for building systems that are skilled at carrying out activities such as: legal action simulation, legal compliance checking, conflicts’ resolution, among others.

In this work, we have assumed a single objective ordering on the objects of the domain, along the lines of the tradition in non-monotonic reasoning and related areas such as conditional logics and belief revision. Of course, one of the drawbacks of having a single ordering is the fact that, within a preferential interpretation, objects are compared in absolute terms, not relative to a context.

Recently, Britz and Varzinczak have proposed a preferential extension of DLs addressing precisely this issue [58]. There, a notion of context based on roles is introduced, which gives rise to multiple preference relations on the set of objects of the domain. With that, it becomes possible to specify that an object $x$ is more normal than $y$ w.r.t. a context $r_1$, whereas $y$ is more typical than $x$ w.r.t. a different context $r_2$.

In the present work, we have decided to stick to the single-preference version of preferential DLs because it is the only one for which a working implementation is available. An extension of DIP to handle multiple preferences forms part of the upcoming tasks in the development of non-monotonic DLs.

Looking forward, this research has shown encouraging results, allowing the theoretical framework to be expanded soon to deal with other principles. This is the case of the “Trifle principle”, which removes any criminal liability if the stolen property is of irrelevant value. This is entirely related to the more general principle of De Minimis Non Curat Lex [59], in which a behavior with extremely low transgression of the law is not classified as illegal. We also plan to provide a separate ontology, capable of performing specific tasks such as the applicability of laws. Thus, we would separate the domain itself from the probable legal actions that can be carried out.

An ongoing work is the development of a prototype, in which an ordinary user can simulate lawsuits in real or fictitious cases through a friendly experience, without bothering with the low formal level of DL. LEGIS (LEGal analysIS) is a web-based front-end system, through which one can make functional and affordable checks carried out by the mapped criminal ontologies. We expect the results obtained so far might improve the juridical understanding of layperson and aid the labor-intensive task of lawsuits performed by professional jurists.

Given that knowledge-based systems aim to support human decision-making, the need for legible descriptions of how the results were inferred becomes indispensable. Therefore, another proposal of future work is the integration of these tasks of reasoning with the research proposed by [60], in order to allow the user to understand the explanations of the inferences made. [60] proposes a conversion method which translates $\text{AFC}$ connection proofs into $\text{AFC}$ sequent proofs. The connection method [61] is an efficient proof system for first-order logic and it already has a variant for reasoning over ontologies written in the DL $\text{AFC}$. However, its proofs are not very readable, which makes interaction with users in general difficult. With the conver-
sion to sequents, a more readable and intelligible representation is obtained, since sequent calculus [62] is essentially a formal logic argument style. The conversion method might be used in practical applications, in areas that employ DL reasoning and generate descriptions on natural language inferences for lay users. The proof conversion can help users understand why a particular situation is characterized as a crime, making its use viable in practice.

Finally, it should be noted that this project comes from a joint effort of cooperation between computer scientists and legal experts. The assumptions made here were the result of a joint analysis between these teams. From this cooperation, once LEGIS is fully operational, we will apply the prototype as a subsidiary system for decisions in local courts.

Acknowledgement

This research is part of the project APQ-0550-1.03/16 (Reconciling Description logic and non-monotonic reasoning in the legal domain), supported by Fundação de Amparo à Ciência e Tecnologia do Estado de Pernambuco (FACEPE), by Institut National de Recherche en Informatique et Automatique (INRIA) and by the Centre National de la Recherche Scientifique (CNRS).

References


W.W. Smari, ed., IEEE Computer Society, Los Alamitos, CA,

Description Logic to Model a Domain Specific Information Retriev
System, in: Database and Expert Systems Applications:
19th International Conference, DEKA 2008, Turin,
Italy, September 1-5, 2008. Proceedings, S.S. Bhownick,
J. King and R. Wagner, eds, Springer Berlin Heidelberg,
85654-2. doi:10.1007/978-3-540-85654-2_17. https://doi.org/
10.1007/978-3-540-85654-2_17.

[20] A.C. Tran, Application of description logic learning in abnor
mal behaviour detection in smart homes, in: The 2015 IEEE
RIVF International Conference on Computing Communica
tion Technologies - Research, Innovation, and Vision for Fu
doi:10.1109/RIVF.2015.7049866.

marization approach based on description logic theory, in:
2008 6th IEEE International Conference on Industrial In
doi:10.1109/INDIN.2008.4618301.

[22] J. Cheng, Z.M. Ma and Y. Wang, Query Answering in
Fuzzy Description Logics with Data Type Support, in: 2010
IEEE/WIC/ACM International Conference on Web Intelligence
doi:10.1109/WI-IAT.2010.81.

vey, in: The Logic of Legal Requirements - Essays on De
feasibility, Oxford University Press, Oxford, United Kingdom,
2013, pp. 11–38.

soning, Preferential Models and Cumulative Logics, Artificial
Intelligence 44(1–2) (1990), 167–207.

ck, Ordered Interpretations and Entailment for Defeasible De
scription Logics, Technical Report, CAIR, CSIR Meraka and UKZN, South

82.

[27] J. Pauwelyn, Resolving conflict in the applicable law, in: Con
flict of norms in public international law: how WTO law re
lates to other rules of international law, Cambridge University

[28] B. Celano, True exceptions: defeasibility and particularism
(2012), 268–287.

[29] L.G. Boomin, Concerning the Defeasibility of Legal Rules, Phi
losophy and Phenomenological Research 26(3) (1966), 371–
378.

perstructure, Version 2.4.1, Object Management Group, 2011.

[31] C.M.D.O. Rodrigues, C. Bezerra, F. Freitas, and I. Oliveira,
Handling Crimes of Omission By Reconciling a Criminal Core

tology and some Applications of it in Business Modeling,
in: CAiSE Workshops, 3, J. Grundspenkis and M. Kirikova,
ed., Faculty of Computer Science and Information Technol
ogy, Riga Technical University, Riga, Latvia, 2004, pp. 129–

[33] G. Guizzardi and G. Wagner, Towards Ontological Founda
tions for Agent Modelling Concepts Using the Unified Foun
dational Ontology (UFO), in: Agent-Oriented Information Sys
tems II: 6th International Bi-Conférence Workshop, AOIS
2004, Riga, Latvia, June 8, 2004, and New York, NY, USA, July
20, 2004, Revised Selected Papers, P. Bresciani, P. Giorgini,
B. Henderson-Sellers, G. Low and M. Winkoff, eds, Springer
doi.org/10.1109/11426714_8.

[34] G. Casini, T. Meyer, K. Moodley, U. Sattler and I. Varzin
ck, Introducing Defeasibility into OWL Ontologies, in: The Se
mantic Web - ISWC 2013, M. Arenas, O. Corcho, E. Simperl,
M. Strohmaier, M. d’Aquin, K. Srinivas, P. Groth, M. Dumon
tier, J. Heflin, K. Thirunarayan and S. Staab, eds, Springer In
3-319-25010-6.

[35] M. Araszkiewicz, Legal Rules: Defeasible or Indefea
sible?, in: Problems of Normativity, Rules and Rule-
Following, M. Araszkiewicz, P. Banaș, T. Gizbert-
Studnicki and K. Pleszka, eds, Springer International
3-319-09375-8. doi:10.1007/978-3-319-09375-8_31.

doi:10.1016/S0004-3702(03)00122-
S000437020300122X.

[37] M. Atienza and J.R. Manero, Rules, Principles and Defeasibility,
in: The Logic of Legal Requirements - Essays on De
feasibility, J.F. Beltrán and G.B. Ratti, eds, Oxford Univer
sity Press, Oxford, United Kingdom, 2012, pp. 238–253, Chap-
ter 14.

[38] R. Reiter, A Logic for Default Reasoning, Artificial Intelli
gence 13(1,2) (1980), 81–132.


[40] D. Nute, Defeasible Logic, in: Handbook of Logic in Artificial
Intelligence and Logic Programming-Nonmonotonic Reasoning
and Uncertain Reasoning, Vol. 3, D.M. Gabbay, J.C. Hog
following, J.A. Robinson, eds, Clarendon Press, Oxford, UK,

[41] G. Governatori, F. Olivieri, S. Scannapieco and M. Cristani,
Superiority Based Revision of Defeasible Theories, in: Seman
tic Web Rules: International Symposium, RuleML 2010, Wash
ington, DC, USA, October 21-23, 2010, Proceedings, M. Dean,
J. Hall, A. Rotolo and S. Tabet, eds, Springer Berlin Heidelberg,
doi:10.1007/978-3-642-16289-3_10. https://doi.org/
10.1007/978-3-642-16289-3_10.

[42] G. Sartor, A Simple Computational Model for Nonmonotonic
and Adversarial Legal Reasoning, in: Proceedings of the 4th
International Conference on Artificial Intelligence and Law,
ICAIL ’93, ACM, New York, NY, USA, 1993, pp. 192–201.

Appendix A. Criminal Types In Portuguese

In Portuguese, the Crimes against property read as follows:

– Furto: Subtrair, para si ou para outrem, coisa alheia móvel. (Art. 155);
– Roubo: Subtrair coisa móvel alheia, para si ou para outrem, mediante grave ameaça ou violência [...] (Art. 157);
– Latrocínio: Se da violência resulta [...] morte, a reclusão é de vinte a trinta anos (Art. 157, § 3º);